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| Name: | |
| Class: | |

# TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2017

# MATHEMATICS

**General Instructions**:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Reference Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

**Total Marks 100**

**Section I**: **10 marks**

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

**Section II**: **90 Marks**

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

**The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.**

## Section I Multiple Choice (10 marks)

**Attempt Question 1 – 10 (1 mark each)**
**Allow approximately 15 minutes for this section**.

**1.** What is the size of the angle subtended at the centre of a circle with diameter 7cm by an arc 3cm long?

A. $25°$ B. $49°$ C. $67°$ D. $133°$

**2.** What is the limiting sum of the series $45 - 15 + 5 - \frac{5}{3} + \cdots$

A. $\frac{135}{2}$ B. $\frac{45}{4}$ C. $\frac{135}{4}$ D. $\frac{-45}{2}$

**3.** Evaluate $\int_{-3}^{2} 3x^2 - 4x + 1\, dx$:

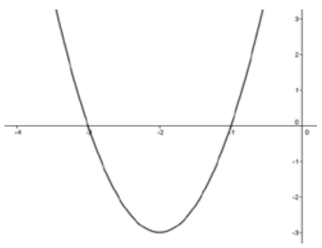
A. $-10$ B. $50$ C. $14$ D. $46$

**4.** $\frac{3+\sqrt{2}}{3-\sqrt{2}} - 5$ is equivalent to:

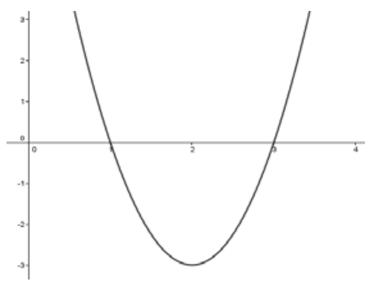
A. $\frac{-21(2+\sqrt{2})}{7}$ B. $\frac{2(23+3\sqrt{2})}{7}$ C. $\frac{6(5\sqrt{2}-8)}{7}$ D. $\frac{6(\sqrt{2}-4)}{7}$

**5.** The graph of the gradient function of $y = x^3 + 6x^2 + 9x$ is:

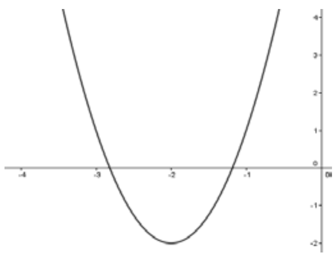
A.



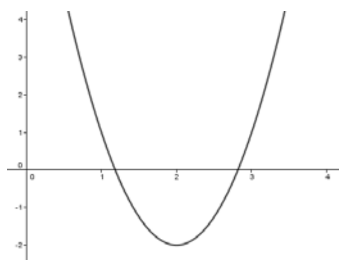
B.



C.



D.



**6.** John has 10 marbles in a bag, 3 of which are green and the rest are yellow. He randomly draws 1 out of the bag, notes its colour and then puts it back. What is the least number of times John will have to draw so that he has over 95% chance of getting a green marble at least once?

A. 8 B. 9 C. 2 D. 3

**7.** Find the locus of a point that is equidistant from the point (0,3) and the line $x = -1$

A. $(y-3)^2 = 2(x+\frac{1}{2})$

B. $x^2 = 8(y+1)$

C. $x^2 = -8(y+1)$

D. $(y-3)^2 = -2(x+\frac{1}{2})$

**8.** A hundred students in a particular cohort are required to choose at least 1 of 2 languages as a subject. There are 55 students who chose Japanese as a subject and 83 students chose Italian as a subject. A student is randomly selected out of the 100, what is the probability of the student doing both languages?

A. $\frac{28}{100}$

B. $\frac{28}{138}$

C. $\frac{38}{100}$

D. $\frac{38}{138}$

**9.** The solution to $|3x-1| = 2x-1$

A. $x = 0$ and $x = \frac{2}{5}$ B. $x = 0$ only C. $x = \frac{2}{5}$ only D. No Solutions

**10.** The displacement $x$ metres of a particle moving in a straight line at time $t$ seconds is given by:

$$x = 2t - 4\log_e(2t+1).$$

Which one of the following statements is false?

A. The acceleration is always positive

B. The initial velocity is $-6ms^{-1}$

C. The particle is initially at the origin

D. As $t \to \infty, x \to \infty$ and $v \to \infty$

## End of Section I

## Section II Total Marks is 90

**Attempt Questions 11 – 16.**
**Allow approximately 2 hours & 45 minutes for this section.**

Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper.

All necessary working must be shown in each and every question.

**Question 11** (15 marks)

a) Find the derivative of:

i. $(x+3)e^{2x}$ (Factorise your answer) 2

ii. $\frac{1+e^x}{ln(x)}$ 2

b) Let $\alpha$ and $\beta$ be the roots of the quadratic equation $3x^2 - 7x + 9 = 0$.

i. Find the value of $\alpha^2 + \beta^2$. 2

ii. Factorise $\alpha^3 + \beta^3$ 1

iii. Hence or otherwise, find the value of $\alpha^3 + \beta^3$ 1

c) A locus is given by the equation $x^2 + 2y = 8x - y^2 - 8$.

i. By first completing the square, describe the above locus geometrically. 3

ii. Find the $x$ and $y$ intercepts (if any) of the locus as exact values. 2

iii. Neatly sketch the locus on a number plane, showing all necessary information. Clearly mark all of the intercepts. 2

**Question 12** (15 marks) Start a new sheet of paper

a) A, B and C are points on the number plane with coordinates (0, 6), (3, 0) and (9, 3) respectively.

i. Plot the information above on a number plane. (You may add further information on the diagram as the question progresses) 1

ii. Find the coordinates of M, the midpoint of AC. 1

iii. Show that the equation of AC is given by $x + 3y - 18 = 0$ and the equation of $l_1$, the perpendicular bisector of AC, is given by $3x - y - 9 = 0$. 4

iv. Find the coordinates of D above AC, such that it lies on $l_1$ and is $\sqrt{10}$ units from AC. 3

v. Show that B, M and D are collinear. 1

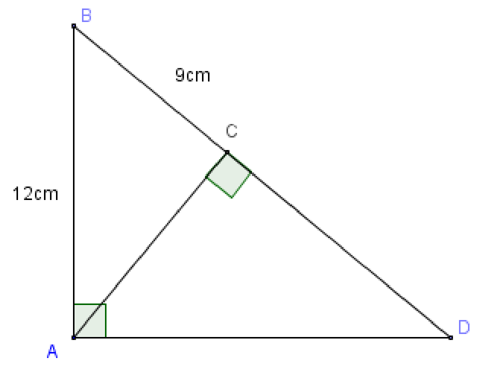
vi. Without finding the actual distance between A,B, C and D, explain why ABCD is a kite. 1

vii. Find the area of ABCD. 3

viii. Find the acute angle $l_1$ makes with the $x$-axis to the nearest degree. 1

**Question 13** (15 marks) Start a new sheet of paper

a) In the diagram, $\Delta ADC$ and $\Delta ABD$ are both right angle triangles, with $BC = 9cm$ and $AB = 12cm$.




i. Prove that $\Delta ABD \ ||| \ \Delta CAD$ 2

ii. Hence find the length of DC and AD. 3

b) Solve $\log_2 x = 3 - \log_2(x - 2)$ 3

c) Solve $3sin^2\theta = sin\theta$ for $0 \leq \theta \leq 2\pi$ (Give your answer in radians) 3

d)

i. Sketch a graph of the region bounded by the curves $y = x^2$ and $y = x^3$, clearly show their points of intersection at (0, 0) and (1, 1). 2

ii. Find the volume of the solid that is formed when the region in part i) rotated about the $x$-axis. 2

**Question 14** (15 marks) Start a new sheet of paper

a) The population P of a small town is decreasing according to the equation $\frac{dP}{dt} = -kP$, where time $t$ is the number of years since the beginning of May 2015 and $k$ is a positive constant.

At the start of May 2015 the town had a population of 2040. However, the population had halved by the beginning of May 2017.

It is given that $P = P_0 e^{-kt}$ is a solution of $\frac{dP}{dt} = -kP$.

i. Find the value of $P_0$. 1

ii. Show that the value of $k$ is $\frac{1}{2} ln2$. 2

iii. The town mayor decrees that when there are only 100 people left then they must all leave. During which month of which year will this happen? 3

b) A particle initially at $x = -2$, moves so that at time $t$ seconds its velocity $v\ m/s$ is given by:

$$v = 2\pi + 2\pi(sin\pi t).$$

i. Find the initial velocity. 1

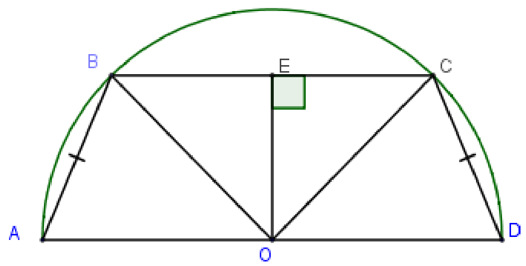
ii. Find the time when the particle is first stationary, and its displacement at that time. 3

iii. Sketch a velocity-time graph for the first 2 seconds. 2

c) Two employees are paid daily in two different schemes. The 1st employee receives a payment that is 3 times more than what he was paid the day before. The 2nd employee receives a payment that is $50 dollars more than what he was paid the day before. Both employees were paid the same amount of money on day 1 and the same amount in total after 8 days of work. How much did they get paid on day 1? Write your answer to the nearest cent. 3

**Question 15** (15 marks) Start a new sheet of paper

a) An isosceles trapezium ABCD is drawn with its vertices on a semi-circle centre O and diameter 20 cm (see diagram). OE is the altitude of ABCD.




i. Prove that $\Delta BOE \equiv \Delta COE$ 2

ii. Hence or otherwise, show that the area of the trapezium ABCD is given by: 3

$$A = \frac{1}{4}(x + 20)\sqrt{400 - x^2}$$

where $x$ is the length of BC.

iii. Hence find the length of BC so that the area of the trapezium ABCD is a maximum. 3

b) An unbiased coin is tossed 3 times, and the upper most face is recorded after each toss.

i. Draw a tree diagram to illustrate all the possible outcomes 1

ii. What is the difference in probability between getting exactly two heads and getting the same outcome for all three tosses? 3

c) Use the trapezoidal rule with 2 sub-intervals to estimate the value of $\int_{-2}^{2} xe^x \, dx$ correct to 1 decimal place. 3

**PLEASE TURN OVER**

**Question 16** (15 marks) Start a new sheet of paper

a) Hercules walks at a speed of 3km/h at a bearing of 030T. Two hours later from the same spot, Xena walks at a speed of 5km/h at a bearing of 160T.

i. Write an expression for the distance Hercules is from the origin t hours after Xena started her walk. 1

ii. Find the distance between Hercules and Xena 8 hours after Xena started her walk. 3

b) A home loan of $\$600\ 000$ is borrowed from a bank at a rate of 6% $p.a.$ on the 1st of January 2017 over 10 years, compounded monthly. Interest is charged at the beginning of each month (including immediately when the loan was taken), while repayments are made at the end of every month. Let M be the amount of each monthly instalment.

Let $I_n$ represent the amount owing after the $n^{th}$ repayment.

i. Show that $I_n = 600000(\frac{201}{200})^n - 200M[\left(\frac{201}{200}\right)^n - 1]$ 3

ii. The loan is to be paid back over 10 years. Show that $M = 6661.23$ 2

iii. The bank reduced its interest rates from 6% $p.a.$ to 5% $p.a.$ on the 1st of January 2022. Find, to the nearest cent, the amount owing just before the rate changes. 1

iv. Denote your answer in part iii. as $'A'$ for this question.

If the repayment stays the same and taking into account the rate change, in which year and month can the loan be expected to be paid off? 3

Hint: Do all your working out in terms of $A$ and $M$, only substitute your answers from ii. and iii. at the end.

v. Due to the rate change in part iii, the bank requires the last payment to only pay off the remaining balance as opposed to the usual instalment amount. How much interest in total will the borrower have paid over the life of the loan? 2

## End of Paper

Student Number MC

1. $l = r\theta$

$\theta = \frac{l}{r} = \frac{3}{3.5} = \frac{6}{7}$ (B)

$\theta$ in degrees $= 49°$

2. $a = 45 \quad r = \frac{-1}{3}$

$S_\infty = \frac{45}{1-(\frac{-1}{3})} = \frac{45}{4/3} = \frac{135}{4}$ (C)

3. $\left[x^3 - 2x^2 + x\right]_{-3}^{2} = (8-8+2) - (-27-18-3)$

$= 2 - (-48)$

$= 50$ (B)

4. $\frac{3+\sqrt{2}}{3-\sqrt{2}} - 5 = \frac{3+\sqrt{2} - 15 + 5\sqrt{2}}{3-\sqrt{2}}$

$= \frac{-12+6\sqrt{2}}{3-\sqrt{2}}$

$= \frac{(-12+6\sqrt{2})(3+\sqrt{2})}{9-2}$

$= \frac{-36-12\sqrt{2}+18\sqrt{2}+12}{7}$

$= \frac{-24+6\sqrt{2}}{7}$ (D)

$= \frac{6(\sqrt{2}-4)}{7}$

Student Number

5. $\frac{dy}{dx} = 3x^2 + 12x + 9$

$= 3(x^2+4x+3)$

$= 3(x+3)(x+1)$ (A)

6. $P(\text{green}) = 3/10 \quad P(\text{yellow}) = 7/10$

$P(\text{at least 1 green}) = 1 - P(\text{All yellow})$

$= 1 - (7/10)^n$

$\therefore$ Solving for $1 - (7/10)^n > \frac{19}{20}$

$(7/10)^n < \frac{1}{20}$

$\ln(7/10)^n < \ln(\frac{1}{20})$

$n \ln(7/10) < \ln(\frac{1}{20})$

$n > \frac{\ln(\frac{1}{20})}{\ln(7/10)}$

$n > 8.39$

$\therefore n = 9.$

(B)

7. $\sqrt{(x-0)^2 + (y-3)^2} = (x+1)$

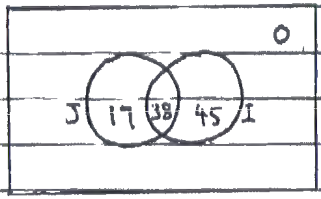
$x^2 + (y-3)^2 = x^2 + 2x + 1$ (A)

$(y-3)^2 = 2x+1$

$(y-3)^2 = 2(x+\frac{1}{2})$

Student Number

8.




$\therefore P(\text{Both}) = \frac{38}{100}$

9. Solve $(3x-1)^2 = (2x-1)^2$

$$9x^2 - 6x + 1 = 4x^2 - 4x + 1$$

$$5x^2 - 2x = 0$$

$$x(5x-2) = 0$$

$$\therefore x = 0 \text{ or } \frac{2}{5}$$

check $x=0$, LHS $= 1$, RHS $= -1$, Incorrect.

$x = \frac{2}{5}$, LHS $= \left|\frac{6}{5} - 1\right| = \frac{1}{5}$, RHS $= \frac{4}{5} - 1$

$= \frac{-1}{5}$ $\therefore$ Incorrect

$\therefore$ No solutions D

10. $x = 2t - 4\ln(2t+1)$ $x(0) = 0$

$v = 2 - \frac{4}{2t+1} \times 2$ $v(0) = -6$

$a > 0$

$v = 2 - \frac{8}{4t+1}$ as $t \to \infty$, $v \to 2$, $a \to 0$.

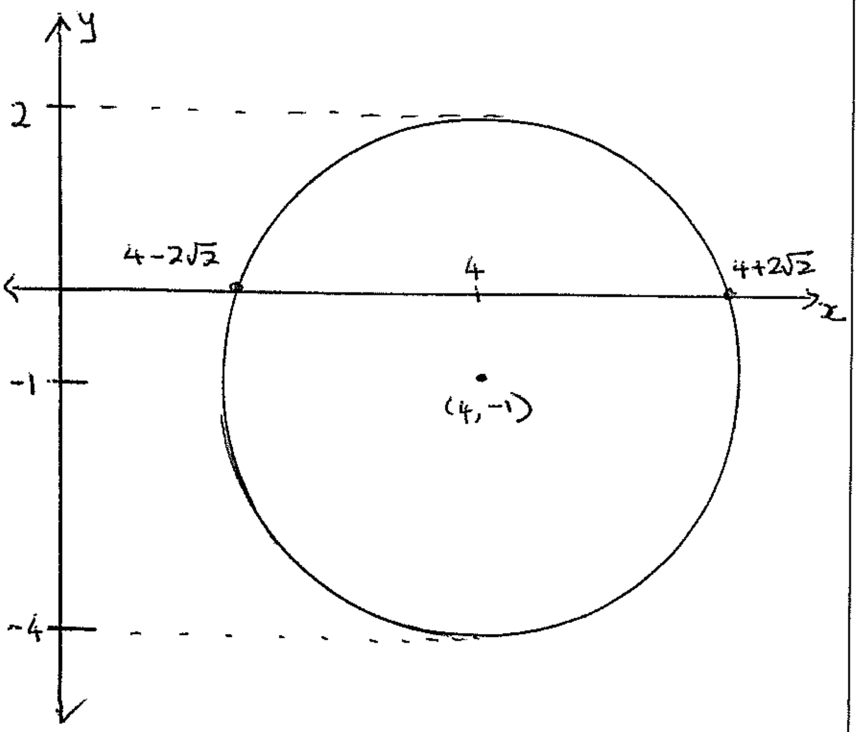
$a = 8(4t+1)^{-2} \cdot 4$ $\therefore$ D

$= \frac{32}{(4t+1)^2}$

## MATHEMATICS: Question 11

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| a) Derivative of<br>(i) $(x+3)e^{2x}$.<br>$\frac{dy}{dx} = (x+3)2e^{2x} + e^{2x}$ ✓<br>$= e^{2x}(2x+6+1)$<br>$= e^{2x}(2x+7)$ ✓ | 2 marks<br>✓ process<br>✓ answer. | let $u = (x+3)$<br>$u' = 1$<br>$v = e^{2x}$<br>$v' = 2e^{2x}$ |
| (ii) $\frac{1+e^x}{\ln x}$<br>$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$<br>$= \frac{\ln x\, e^x - (1+e^x)\frac{1}{x}}{(\ln x)^2}$<br>$= \frac{x e^x \ln x - 1 - e^x}{x(\ln x)^2}$ | 2 marks<br>✓ sub in correct formula<br>✓ simplifying<br>Had to eliminate fractions from numerator to simplify | $u = 1+e^x$<br>$u' = e^x$<br>$v = \ln x$<br>$v' = \frac{1}{x}$<br>- accepted<br>$\frac{e^x(x\ln x - 1) - 1}{x(\ln x)^2}$ |
| b (i) $3x^2 - 7x + 9 = 0$<br>$\alpha+\beta = \frac{-b}{a} = \frac{7}{3}$<br>$\alpha\beta = \frac{c}{a} = 3$<br>(i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$<br>$= \left(\frac{7}{3}\right)^2 - 2(3)$<br>$= \frac{49}{9} - 6$<br>$= -5/9$ | 2 marks<br>✓ simplifying $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$<br>✓ getting $-5/9$ as answer | well done |

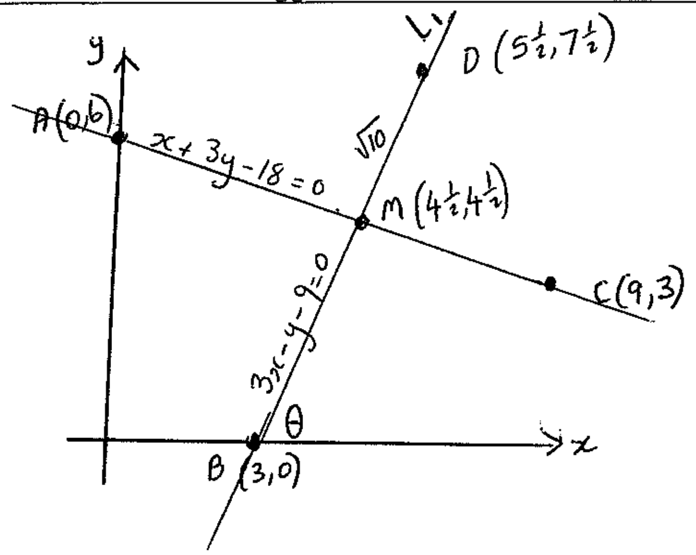
| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| b(ii) Factorise $\alpha^3+\beta^3$<br>$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ ✓ | 1 mark. | |
| (iii) Value $\alpha^3+\beta^3$<br>$\alpha^3+\beta^3=\left(\frac{7}{3}\right)\times\left(\frac{-5}{9}-3\right)$<br>$=\frac{7}{3}\times\frac{-32}{9}$<br>$=-\frac{224}{27}$ ✓ | 1 mark | |
| c) $x^2+2y=8x-y^2-8$<br>(i) $x^2-8x+y^2+2y+8=0$<br>$x^2-8x+16+y^2+2y+1=-8+16+1$<br>$(x-4)^2+(y+1)^2=9$<br>locus is a circle with centre $(4,-1)$ and radius 3 | 3 marks)<br>✓ correct completion of square<br>✓ getting $(x-4)^2+(y+1)^2=9$<br>✓ writing locus as circle with radius. | |
| (ii) y intercept put $x=0$<br>$(x-4)^2+(y+1)^2=9$<br>$4^2+(y+1)^2=9$<br>$(y+1)^2=-7$<br>$\therefore$ No y-intercepts ✓<br>$x$-intercept put $y=0$<br>$(x-4)^2+(0+1)^2=9$<br>$(x-4)^2+1=9$<br>$(x-4)^2=8$<br>$x-4=\pm\sqrt{8}$<br>$x=4\pm\sqrt{8}$ or $4\pm2\sqrt{2}$ ✓ | 2 marks<br>✓ there is no y-intercepts<br>✓ x-intercepts | |

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
|  | 2 marks | ✓ shape<br>centre point<br>✓ x-intercept<br>and maximum/minimum<br>y values<br>ie 2 and -4 |

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|

(a)

(i)




1

A, B, C labeled
$x$-$y$ axis labeled
axis roughly to scale.

NOTE! Add any other features as you progress through the question.

(ii) $M = \left(\frac{9+0}{2}, \frac{6+3}{2}\right)$

$M = (4\frac{1}{2}, 4\frac{1}{2})$. (midpoint of AC).

1

(iii) To find equation of AC

$m_{AC} = \frac{3-6}{9-0} = \frac{-3}{9} = -\frac{1}{3}$

1

Eqn of AC: using $y = mx + b$

$y = -\frac{1}{3}x + 6$

$x + 3y - 18 = 0$

1

Alternatively,

$\frac{y-6}{x-0} = \frac{6-3}{0-9}$

$\frac{y-6}{x} = -\frac{1}{3}$

$3y - 18 = -x$

$x + 3y - 18 = 0$

Note! You needed to show all steps clearly as the answer was given. or you do not get the mark.

Since $L_1$ is perpendicular then $m_1 m_2 = -1$. $L_1$ passes through $M\left(\frac{9}{2}, \frac{9}{2}\right)$

$\therefore m_{L_1} = 3$

1

$\therefore$ Eqn of $L_1$: $y - \frac{9}{2} = 3\left(x - \frac{9}{2}\right)$

$2y - 9 = 6\left(x - \frac{9}{2}\right)$

$2y - 9 = 6x - 27$

$6x - 2y - 18 = 0$

$3x - y - 9 = 0$

1

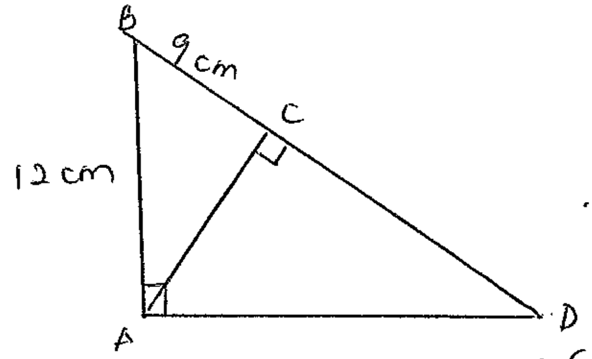
Must show working because once again the answer is given.

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (IV) Let the coordinates of D be $(x,y)$<br>The perpendicular distance of D from AC $(x+3y-18=0)$ is $\sqrt{10}$.<br>$a=1$, $b=3$, $c=-18$<br>$\therefore \sqrt{10} = \dfrac{\lvert x+3y-18\rvert}{\sqrt{1+9}}$<br>$10 = \lvert x+3y-18\rvert$ | 1 | For using perpendicular distance of point from a line formula & substituting correctly. |
| $x+3y-18=10$ OR $x+3y-18=-10$<br>$x+3y-28=0$ $\quad$ $x+3y-8=0$<br>$y=-\frac{x}{3}+\frac{28}{3}$ $\quad$ $y=-\frac{x}{3}+\frac{8}{3}$<br>Since D is above the line then $x+3y-28=0$ is the line $\sqrt{10}$ units from AC. To find the point D, solve<br>$x+3y-28=0$ — (1) and $l_1$ simultaneously.<br>$3x-y-9=0$ — (2) | | |
| (1) × 3 $\quad 3x+9y-84=0$ — (1)×3<br>$-l_1 \quad -(3x-y-9=0)$ — (2)<br>$10y-75=0$<br>$y=\frac{75}{10}$<br>$y=\frac{15}{2}$ or $7\frac{1}{2}$ | 1 | |
| When $y=\frac{15}{2}$ substitute into (1)<br>$x+3\left(\frac{15}{2}\right)-28=0$<br>$x+\frac{45}{2}-28=0$<br>$x=\frac{11}{2}$ or $5\frac{1}{2}$<br>$\therefore D=\left(5\frac{1}{2}, 7\frac{1}{2}\right)$ | 1 | |

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (V) To show B, M & D are collinear substitute B into eqn $l_1$. $B=(3,0)$ | | |
| $3(3)-(0)-9=0$ | | |
| $\therefore B(3,0)$ lies on the line $l_1$. | | |
| $\therefore$ B, M and D are collinear. | 1 | |
| Alternatively | | |
| $m_{BM}=\frac{4\frac{1}{2}-0}{4\frac{1}{2}-3}=\frac{4\frac{1}{2}}{1\frac{1}{2}}=3$    $m_{MD}=\frac{7\frac{1}{2}-4\frac{1}{2}}{5\frac{1}{2}-4\frac{1}{2}}=\frac{3}{1}=3.$ | | |
| Since $m_{BM}=m_{MD}$ then B, M and D are collinear. | Alternate 1 | |
| (VI) Since $l_1$ is the perpendicular bisector of AC, then ABCD is a kite. (One diagonal is the perpendicular bisector of the other). | 1 | Note! It is not sufficient to say the diagonals are perpendicular. Must also mention bisector. |

| Suggested Solutions | Marks | Marker's Comments |
| --- | --- | --- |
| (vii) $d_{AC} = \sqrt{(9-0)^2+(3-6)^2}$ $= \sqrt{81+9}$ $= \sqrt{90}$ | 1 | |
| $d_{BD} = \sqrt{\left(3-\frac{11}{2}\right)^2+\left(0-\frac{15}{2}\right)^2}$ $= \sqrt{\frac{25}{4}+\frac{225}{4}}$ $= \sqrt{\frac{250}{4}}$ $= \frac{5\sqrt{10}}{2}$ | 1 | Alternatively you could find the areas using triangles. eg $AC = 3\sqrt{10}$, $BM = \frac{15}{\sqrt{10}}$ |
| Area $ABCD = \frac{1}{2}\times AC \times BD$ $= \frac{1}{2}\times\sqrt{90}\times\frac{5\sqrt{10}}{2}$ $= \frac{1}{2}\times 3\sqrt{10}\times\frac{5\sqrt{10}}{2}$ $= \frac{150}{4}u^2$ OR $37\frac{1}{2}u^2$ | 1 | Area = Area $\triangle ADC$ + Area $\triangle ABC$ $= \frac{1}{2}\times 3\sqrt{10}\times\sqrt{10} + \frac{1}{2}\times 3\sqrt{10}\times\frac{15}{\sqrt{10}}$ $= 15 + \frac{45}{2}$ $= 37\frac{1}{2}$ |
| (viii) $\tan\theta = \frac{7\frac{1}{2}}{2\frac{1}{2}}$ $\left(\text{or } \frac{4\frac{1}{2}}{1\frac{1}{2}}\right)$ $\tan\theta = 3$ $\theta = \tan^{-1}3$ $= 72^\circ$ (nearest degree) | 1 | Note! Answer in radians, <u>no</u> marks. (Make sure your calculator is in correct mode). $71^\circ$ accepted if $\tan\theta = 3$ is shown. |

**MATHEMATICS: Question** 13 1/4

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| a i) In $\Delta ABD$ and $\Delta CAD$ | | |
|  | | |
| $\angle BAD = \angle ACD = 90^\circ$ (given) | } — 1 | |
| $\angle D$ is common | | |
| $\therefore \Delta ABD \ \vert\vert\vert \ \Delta CAD$ (equiangular) | — 1 | |
| ii) $9^2 + AC^2 = 12^2$ (pythagoras' Theorem) | | |
| $AC = 144 - 81$ | | |
| $= \sqrt{63}$ $(AC > 0)$ * | | please state |
| $= 3\sqrt{7}$ | | |
| $\dfrac{\Delta ABD}{\Delta CAD} = \dfrac{AB}{CA} = \dfrac{BD}{AD} = \dfrac{AD}{CD}$ corresponding sides in similar triangles are * in same ratio | | — some students wrote are equal |
| $\dfrac{12}{\sqrt{63}} = \dfrac{9+CD}{AD} = \dfrac{AD}{CD}$ | | |
| $AD = \dfrac{12CD}{\sqrt{63}}$ —— i | 1 — | correct reasons |
| $AD^2 = CD(9+CD)$ —— ii | 2. — | to find AC, AD, CD |
| sub i into ii | | |
| $\left(\dfrac{12CD}{\sqrt{63}}\right)^2 = 9CD + CD^2$ —— iii | | |
| $\dfrac{144CD^2}{63} = 9CD + CD^2$ | | |

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**MATHEMATICS: Question……..** 2/4

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|

$144\,CD^2 = 567\,CD + 63\,CD^2$

$81\,CD^2 = 567\,CD$

$CD = 0$ or $7$ $(CD > 0)$

$\therefore CD = 7$

sub into eqn i

$AD = \dfrac{12(7)}{\sqrt{63}}$

$= \dfrac{84}{\sqrt{63}}$ or $\dfrac{28}{\sqrt{7}}$ or $4\sqrt{7}$

or $10.58$ or $\sqrt{112}$.

OTHER METHODS.

1) trig

$\cos B = \dfrac{9}{12}$

$\therefore B = 41.41^\circ$

$\angle D = 90 - 41.41$

$= 48.59^\circ$

$\sin 48.59 = \dfrac{AC}{AD}$

$AD = \dfrac{\sqrt{63}}{\sin 48.59}$

$= 10.58$.

$AD^2 + AB^2 = BD^2$ (pythagoras theorem)

$10.58^2 + 12^2 = BD^2$

$BD = 15.998$

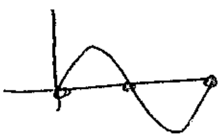
$CD = BD - BC$

$= 15.998 - 9 = 7$ or $6.998$

Marker's Comments:

* some used sine rule
* some used pythagoras all the way through.

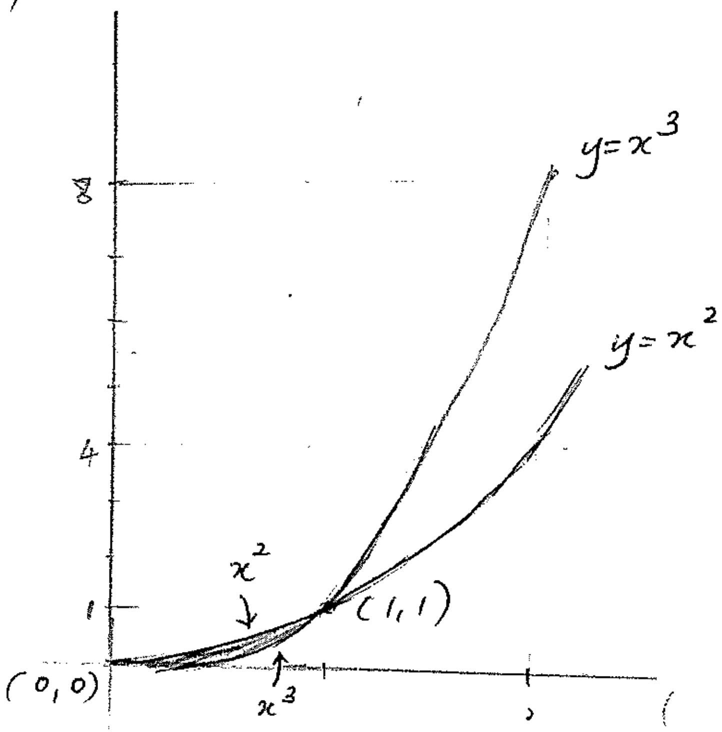
**MATHEMATICS: Question** 13 3/4

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| b) $\log_2 x = 3 - \log_2(x-2)$ | | |
| $\log_2 x = \log_2 2^3 - \log_2(x-2)$ | 1 | |
| $\log_2 x = \log_2\left(\frac{8}{x-2}\right)$ | | |
| $x = \frac{8}{x-2}$ | | |
| $x^2 - 2x - 8 = 0$ | | |
| $x \quad -4$, $x \quad +2$   $(x-4)(x+2) = 0$ | | |
| $x = -2$ or $4$ | 1 | |
| but $x > 0$ $\therefore x = 4$ only | 1 | |
| c) $3\sin^2\theta = \sin\theta$ for $0 \leq \theta \leq 2\pi$ | | ⋆ DO NOT divide the eqn by $\sin\theta$. |
| $3\sin^2\theta - \sin\theta = 0$ | | |
| $\sin\theta(3\sin\theta - 1) = 0$ | 1 | |
| $\sin\theta = 0$   $3\sin\theta = 1$ | | |
| $\sin\theta = \frac{1}{3}$ | | |
| $\therefore \theta = 0, \pi, 2\pi$ | 1 | |
| $\theta = 0.34, 2.8$ | 1 | |
|  $\therefore \theta = 0, 0.34, 2.8, \pi, 2\pi$ | | |

MATHEMATICS: Question B ........ 4/4

| Suggested Solutions | Marks Awarded | Marker's Comments |
| --- | --- | --- |
| | | |

d) i)




1 — shape
1 — positioning correctly.

* – some students had the graph of $x^3$ higher than $x^2$ in the bounded region.
– need not sketch the left side
* must show intersection pts.

ii) $V = \pi \int_0^1 (x^2)^2 - (x^3)^2$

$= \pi \left[ \int_0^1 x^4 - \int_0^1 x^6 \right]$

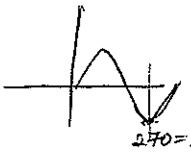
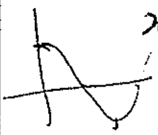
$= \pi \left( \left[ \frac{x^5}{5} \right]_0^1 - \left[ \frac{x^7}{7} \right]_0^1 \right)$

$= \frac{2\pi}{35} u^3$

mistakes!
$(x^2 - x^3)^2$
$(x^3)^2 = x^9$

| MATHEMATICS: Question..14.. | | 1/3 |
|---|---|---|
| **Suggested Solutions** | **Marks Awarded** | **Marker's Comments** |
| a)i) $P = P_0 e^{-kt}$ $\quad \frac{dP}{dt} = -kP_0 e^{-kt}$ <br> $= -kP$ <br> when $t=0$, $P = 2040$. <br> $\therefore 2040 = P_0 e^{-0}$ <br> $\therefore P_0 = 2040$. | 1 | show some working please! |
| ii) $t=2$ $\quad P = 1020$. <br> $1020 = 2040 e^{-2k}$. <br> $e^{-2k} = \frac{1}{2}$ <br> $e^{2k} = 2$. <br> $2k = \ln 2$ <br> $k = \frac{1}{2}\ln 2$ | | |
| iii) $P = 100$ <br> $100 = 2040 e^{-\frac{1}{2}\ln 2(t)}$ <br> $\frac{5}{102} = e^{-\frac{t}{2}\ln 2}$ <br> $\frac{t}{2}\ln 2 = \ln\frac{5}{102}$ <br> $t = 8.7$ years. | 1 <br> 1 | |

**MATHEMATICS: Question 14** $2/3$

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| $= 8$ years and $0 \cdot 7 \times 12$ | | |
| $8 \cdot 4$ months | | |
| $2015 + 8 = 2023$ | | |
| may $+ 8 \cdot 4$ mths $=$ Jan $\quad 0 \cdot 4 \times 31 \approx 12 \cdot 4$ | | |
| $\therefore$ During January 2024 | 1 | |
| b) i) when $t = 0$ for initial condition | | |
| $\therefore v = 2\pi + 2\pi(\sin \pi(0))$ | | |
| $= 2\pi$ m/s | | |
| ii) first stationary $v = 0$. | | |
| $2\pi + 2\pi \sin \pi t = 0$ | | |
|   | | |
| $\sin \pi t = -1$ | | |
| $\pi t = \frac{3\pi}{2}$ | | |
| $t = \frac{3}{2}$. | 1 | |
|  | | |
| $x = \int 2\pi + 2\pi \sin \pi t$ | | |
| $= 2\pi t - 2 \cos \pi t + C$ | | |
| when $t = 0$, $x = -2$ | | |
| $-2 = 0 - 2(1) + C$ | | |
| $\therefore C = 0$ | | |
| $\therefore x = 2\pi t - 2\cos \pi t$ | 1 | |
| when $t = 3/2$ | | |
| $x = 2\pi(3/2) - 2\cos \pi\left(\frac{3}{2}\right)$ | | |
| $= 3\pi - 0$ | | |
| $= 3\pi$ metres | 1 | |

MATHEMATICS: Question 14 3/3

| Suggested Solutions | Marks Awarded | Marker's Comments |
| --- | --- | --- |
| | | |

b)ii) we know $t=0$, $v=2\pi$

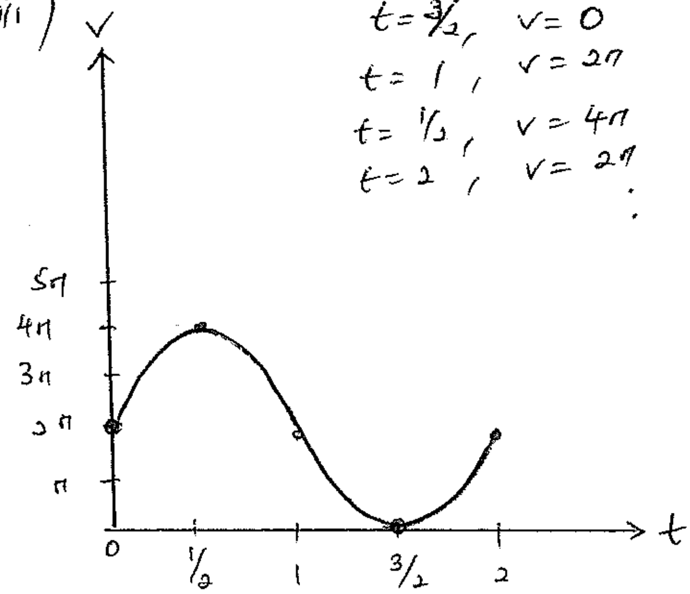
$t=\frac{3}{2}$, $v=0$

$t=1$, $v=2\pi$

$t=\frac{1}{2}$, $v=4\pi$

$t=2$, $v=2\pi$

⋮




c. Employee 1

$T_1$ $a = x$ $T_2 = 3x$ $T_3 = 9x$

$T_n = ar^{n-1}$

$T_8 = x3^7$

$= 2187x$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_8 = \frac{x(3^8-1)}{3-1}$

$= 3280x$

Employee 2

$T_1, a = x$ $T_2 = x+50$

$\therefore d = 50$ $n = 8$

$T_n = a+(n-1)d$

$T_8 = x+(7)50$ — 1

$= x+350$

$S_n = \frac{n}{2}(a+l)$

$S_8 = \frac{8}{2}(x+(x+350))$

$= 4(2x+350)$

$= 8x+1400$ — 1

Equate both

$3280x = 8x + 1400$

$3272x = 1400$

$x = 0.43$ (2 d.p) — 1

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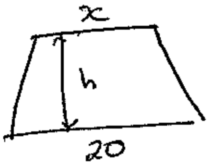
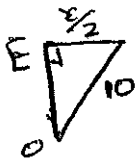
## MATHEMATICS: Question 15

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| a(i) Prove $\triangle BOE \equiv \triangle COE$<br>In $\triangle BOE$ and $\triangle COE$<br>$\angle CEO = \angle BEO = 90^\circ$ (given)<br>EO is common<br>$BO = CO$ (radii of a circle)<br>$\therefore \triangle BOE \equiv \triangle COE$ (RHS) | 2 marks | Accepted SSS<br>- Circle property perpendicular line from centre of circle bisects the chord.<br>* For SAS only accepted if mentioned the above circle property |
| (ii)<br><br>let BC be $x$<br>Area of trapezium $= \frac{1}{2}(a+b)h$<br>$= \frac{1}{2}(x+20) \times EO$ | ✓ | |
| length of EO<br><br>$EO^2 + \left(\frac{x}{2}\right)^2 = 10^2$<br>$EO^2 = 100 - \frac{x^2}{4}$<br>$EO^2 = \frac{400-x^2}{4}$<br>$EO = \frac{\sqrt{400-x^2}}{2}$ $(EO>0)$ | ✓ | |
| $\therefore A = \frac{1}{2}(x+20) \times \frac{\sqrt{400-x^2}}{2}$<br>$= \frac{1}{4}(x+20)\sqrt{400-x^2}$ | ✓ | |

| (iii) Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|

length of BC so that area is maximum

$$\therefore \frac{dA}{dx} = \frac{1}{4}(x+20)\cdot\frac{1}{2}(400-x^2)^{-\frac{1}{2}}\cdot(-2x) + \sqrt{400-x^2}$$

$$= \frac{1}{4}\left[\frac{-x(x+20)}{\sqrt{400-x^2}} + \sqrt{400-x^2}\right]$$

$$= \frac{1}{4}\left[\frac{-x^2-20x+400-x^2}{\sqrt{400-x^2}}\right]$$

$$= \frac{1}{4}\left[\frac{-2x^2-20x+400}{\sqrt{400-x^2}}\right]$$

$$= \frac{1}{2}\left[\frac{-x^2-10x+200}{\sqrt{400-x^2}}\right] \quad ✓$$

Marker's comments:

$u = x+20$ so $u' = 1$

$v = \sqrt{400-x^2}$

$\therefore v' = \frac{1}{2}(400-x^2)^{-\frac{1}{2}} \times -2x$

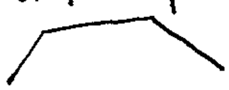
$= \frac{-x}{\sqrt{400-x^2}}$

$\frac{dA}{dx} = 0$

$$-x^2 - 10x + 200 = 0$$
$$x^2 + 10x - 200 = 0$$
$$(x+20)(x-10) = 0$$
$$x = -20 \text{ or } 10$$
$$\therefore x = 10 \text{ only as } x > 0 \quad ✓$$

Test

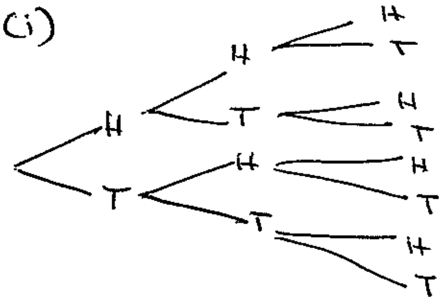
| $x$ | 9 | 10 | 11 |
|---|---|---|---|
| $\frac{dA}{dx}$ | 0.81 | 0 | -0.93 |



$\therefore$ maximum at $x = 10$

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|

b(i)




✓

(ii) P(2 heads) $= \frac{3}{8}$ ✓

P ( same outcomes) $= \frac{2}{8}$ or $\frac{1}{4}$ ✓

Difference $= \frac{1}{8}$ ✓

(c) Trapezoidal Rule

NB 2 sub-intervals only

| | | |
|---|---|---|
| $-2$ | $0$ | $2$ |
| $-2e^{-2}$ | $0$ | $2e^{2}$ |

✓

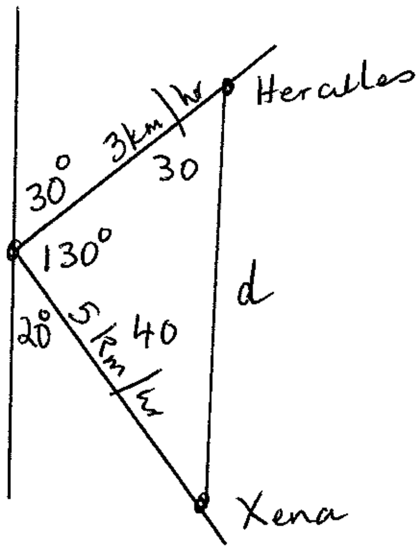
$$A = \frac{2-0}{2}\left[-2e^{-2} + 2(0) + 2e^{2}\right]$$

$$= 1\left(2e^{-2} + 2e^{2}\right) \checkmark$$

$$= -4e^{-2} + 4e^{4}$$

$$\simeq 14.5 \text{ (to 1 dp)} \checkmark$$

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| a) | | |




When Xena starts, Hercules has been walking $(t+2)$ hours.

(i) $D_H = 3 \times (t+2)$ ①

$= 3t + 6$

(ii) When Xena walks for 8 hours, Hercules has been walking for 10 hours.

$\therefore$ When $t = 8$

$D_{xena} = 5 \times 8 = 40\text{km}$ ①

$D_{Herc} = 3 \times 10 = 30\text{km}$

Let d be distance between Xena & Hercules.

$d^2 = 30^2 + 40^2 - 2 \times 30 \times 40 \times \cos 130°$ ①

$= 4042.69$

$d = 63.58\text{ km} \quad d > 0$ ①

Note! if $Hx^2 = 58.15$ then your calculator is in radians & answer is wrong!!

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|

(b) $P = \$600000$ $6\%$ pa

Compounded monthly $r = \frac{.06}{12}$

$= \frac{1}{200}$ (OR 0.005)

After 1st month

$I_1 = 600000 \times \left(1 + \frac{1}{200}\right)^1 - M$ 1 *

$= 600000 \times \left(\frac{201}{200}\right)^1 - M$

$I_2 = \left[600000 \left(\frac{201}{200}\right)^1 - M\right]\left(\frac{201}{200} - M\right)$

$= 600000 \left(\frac{201}{200}\right)^2 - M\left(\frac{201}{200}\right) - M$

$I_3 = 600000 \left(\frac{201}{200}\right)^3 - M\left(\frac{201}{200}\right)^2 - M\left(\frac{201}{200}\right) - M$

$\vdots$

$I_n = 600000 \left(\frac{201}{200}\right)^n - M\left[\left(\frac{201}{200}\right)^{n-1} + \left(\frac{201}{200}\right)^{n-2} + \cdots + \frac{201}{200} + 1\right]$ 1 *

$= 600000 \left(\frac{201}{200}\right)^n - M\left[1 + \frac{201}{200} + \left(\frac{201}{200}\right)^2 + \cdots + \left(\frac{201}{200}\right)^{n-1}\right]$

This is a GP where $a = 1$ & $r = \frac{201}{200}$

$\therefore S_n = 1 \frac{\left(\left(\frac{201}{200}\right)^n - 1\right)}{\frac{1}{200}} = 200\left(\left(\frac{201}{200}\right)^n - 1\right)$ 1 *

$I_n = 600000 \left(\frac{201}{200}\right)^n - M\left(200\left(\frac{201}{200}\right)^n - 1\right)$



$= 600000 \left(\frac{201}{200}\right)^n - 200M\left(\left(\frac{201}{200}\right)^n - 1\right)$

This is given so you MUST show clear working towards this.

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (ii) 10 years = 120 months | | |
| $\therefore t = 120$ | | |
| Loan is paid off when $I_n = 0$ | | |
| ie $0 = 600000\left(\frac{201}{200}\right)^{120} - 200M\left(\left(\frac{201}{200}\right)^{120} - 1\right)$ | | 1 * |
| $200M = \dfrac{600000\left(\frac{201}{200}\right)^{120}}{\left(\frac{201}{200}\right)^{120} - 1}$ | 1 * | OR $M = \dfrac{3000\left(\frac{201}{200}\right)^{120}}{\left(\frac{201}{200}\right)^{120} - 1}$ |
| $200M = \dfrac{1091638.04}{0.819396734}$ | | |
| $200M = 1332246.023$ | | (this answer was given, so explanation working needed to be clear to get the marks) |
| $M = \$6661.23$ | | |
| $\therefore$ Monthly repayments are $6661.23. | | |
| (iii) 1st January 2022 is exactly 5 years (or 60 months). | | |
| $I_{60} = 600000\left(\frac{201}{200}\right)^{60} - \$6661.23 \times 200\left[\left(\frac{201}{200}\right)^{60} - 1\right]$ | | |
| $= 809310.0915 - 464754.2203$ | | |
| $= \$344555.87$ | 1 * | |
| This is the amount owing before the rate change. | | |

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|

(iv) $A = \$344\,555 \cdot 87$

$M = \$6661 \cdot 23$ (monthly repayment).

Let $P_n$ be the amount owing after $n$ repayments.

New interest rate $r = \frac{5\%}{12}$ per month

$= \frac{0 \cdot 05}{12}$

$= \frac{1}{240}$

1 *

$$P_1 = A \times \left(1 + \frac{1}{240}\right)^1 - M$$

$$P_2 = \left(A \times \left(\frac{241}{240}\right)^1 - M\right)\left(\frac{241}{240}\right) - M$$

$$= A\left(\frac{241}{240}\right)^2 - M\left(\frac{241}{240}\right) - M$$

$$P_3 = A\left(\frac{241}{240}\right)^3 - M\left(\frac{241}{240}\right)^2 - M\left(\frac{241}{240}\right) - M$$

$\vdots$

$$P_n = A\left(\frac{241}{240}\right)^n - M\underbrace{\left(1 + \frac{241}{240} + \left(\frac{241}{240}\right)^2 + \dots + \left(\frac{241}{240}\right)^{n-1}\right)}_{\text{this is a GP where}}$$

this is a GP where

$a = 1$

$r = \frac{241}{240}$

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| $P_n = A\left(\frac{241}{240}\right)^n - M\left(\frac{1\left(\frac{241}{240}\right)^n - 1}{\frac{1}{240}}\right)$ | | |
| $= A\left(\frac{241}{240}\right)^n - M\left(240\left[\left(\frac{241}{240}\right)^n - 1\right]\right)$ | | |
| $= A\left(\frac{241}{240}\right)^n - 240M\left(\left(\frac{241}{240}\right)^n - 1\right)$ | | |
| The loan is paid off when $P_n = 0$ | | |
| ie $A\left(\frac{241}{240}\right)^n - 240M\left(\left(\frac{241}{240}\right)^n - 1\right) = 0$ | 1* | |
| $A\left(\frac{241}{240}\right)^n = 240M\left(\left(\frac{241}{240}\right)^n - 1\right)$ | | |
| $A\left(\frac{241}{240}\right)^n = 240M\left(\frac{241}{240}\right)^n - 240M$ | | |
| $A\left(\frac{241}{240}\right)^n - 240M\left(\frac{241}{240}\right)^n = -240M$ | | |
| $\left(\frac{241}{240}\right)^n (A - 240M) = -240M$ | | |
| $\left(\frac{241}{240}\right)^n = \frac{-240M}{A - 240M}$ | | |
| $n \ln\left(\frac{241}{240}\right) = \ln\left(\frac{-240M}{A-240M}\right)$ | | |

$$\frac{\ln\left(\frac{-240M}{A-240M}\right)}{\ln\left(\frac{241}{240}\right)}$$

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| Substitute $A = 344555{\cdot}87$ & $M = \$6661{\cdot}23$ <br> $n = 58{\cdot}37$ <br><br> $\therefore$ by the 59th payment the loan has been paid off. <br> This is 4yrs & 11 months after January 2022 <br> ie November 2026. | 1 * | |
| (v) There are 118 payments (9 yrs & 10mths) of \$6661·23 + the last payment in December 2026. <br><br> $\therefore$ Repayments $= 118 \times \$6661{\cdot}23$ <br> $= \$786025{\cdot}14$. <br><br> After $P_{58} = A\left(\frac{241}{240}\right)^{58} - 240M\left(\left(\frac{241}{240}\right)^{58} - 1\right)$ <br> where $A = \$344555$ & $M = \$6661{\cdot}23$ <br><br> $P_{58} = \$2513{\cdot}83$ <br> This means after the 2nd last payment there is only \$2513·83 left to pay. <br> $\therefore$ the last payment needs to be <br> $\$2513{\cdot}83 \times \left(\frac{241}{240}\right)$ <br> $= \$2524{\cdot}30$ <br> $\therefore$ Total amount paid <br> $= 118 \times 6661{\cdot}23 + 2524{\cdot}30$ | 1 * | |

$= \$788549{\cdot}44$



$\therefore$ Total interest $= \$788549{\cdot}44 - 600000$ (1) *

$= \$188549{\cdot}44$